International Instructional Conference:
Langlands and Geometric Langlands Program
2007, Guangzhou, China

1. Sums of L-functions and applications

Solomon Friedberg

If π is an automorphic representation of $GL(r)$, then the standard Langlands L-function is of the form $L(s,\pi) = \sum a(n)Nn^{-s}$ (N=absolute norm relative to the base field one is working over). If chi is a Hecke character then one may form the twisted Langlands L-function $L(s,\pi,\chi) = \sum a(n)\chi(n)Nn^{-s}$. It is natural to study the family of such functions as chi varies. In particular, let us assume that chi ranges over characters of order exactly M and the base field contains a full set of M-th roots of unity. In this talk I describe a body of work related to this problem, in which one introduces a second complex variable w and forms the sum sum $L(s,\pi,\chi_m)Nm^{-w}$, where m is a parameter indexing the character. Surprisingly, in certain cases this function has continuation in w beyond the region of absolute convergence, and this has implications for the distribution of these L-functions.

2. Weyl Group Multiple Dirichlet Series

Solomon Friedberg

The Langlands-Shahidi method is an important method for studying Langlands L-functions. It is based on the computation of the Whittaker coefficients of Eisenstein series: these Whittaker coefficients can be expressed in terms of products of certain L-functions and this, together with properties of the Eisenstein series, can be used to obtain information about the L-functions themselves. In this talk I describe aspects of the situation when the Eisenstein series in question is on a cover of a classical group, that is, a metaplectic group. This leads to the study of new functions of several variables, Weyl group multiple Dirichlet series, which turn out to have a rich number-theoretic and combinatorial structure.